A PLAN FOR TESTING METHODS OF TEACHING SECONDARY MATHEMATICS

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A friend of mine said the other day: "Mr. Y——— ceases to think about a question so soon as he learns the attitude of his favorite teacher on the question. His final argument in defense of a position is that Mr. Z——— thinks so and so about it."

In a gathering of teachers of secondary mathematics one man argued that the heuristic method is best in teaching mathematics because he had found it best; another asserted with equal emphasis that the method of direct exposition is the best, and for substantially the same reason. A third said that he had found a combination of the two methods better than either one alone; the heuristic for mastering the details of the subject being taught, and the expository for recapitulating its principles with a view to giving a mastery of the subject as a whole. The situation was left in this unsatisfactory status.

In all such gatherings as the one alluded to there will be found no lack of appreciation of such sentiments as the following: "In the beginning with reading, writing, spelling, etc., the statement of the teacher is accepted as authority. In mathematics, personal authority sinks so completely out of sight that the child may even correct the teacher sometimes"; and this: "Rightly taught, mathematics has for one of its chief virtues its value in assisting one to think things out for himself and so delivering him from the yoke of authority"; and finally, even this: "Mathematics is the one school subject in which opinion, tradition, and authority must subordinate themselves to reason."

Can it be that the very spirit and essence of mathematical science fail to carry over into its methods of presentation? Must the representatives of this subject, whose very genius expunges all empiricism and mere authority, acquire and defend their point of view as to its classroom methodology solely on grounds of authority and empiricism? Would it not seem that the spirit of *sufficient*

reason, which pervades all mathematical study worthy of the name, should carry over into methods of teaching and learning mathematics? It is hardly necessary to say that the vital question about the validity of a new method is not at all who believes in it, or has found it the most workable. It is such questions as these: Does it convince the reason? Is it demonstrably correct in its own right? Does it overstress certain important teaching values, and understress, or entirely or too greatly neglect other equally important values? It is not at all the question: Does my good friend believe in it? Is it not high time to formulate more objective and more convincing means of gauging the results of methods of teaching mathematics than have hitherto been devised? Is it not possible so to gather and to organize the evidences of classroom efficiency that a teacher's mind may be convinced at least to the point of a willingness to put a proposed method to test, with some promise of being able actually to measure the type and the degree of excellence, or lack of it, that belong to the method? In short, have not teachers of secondary mathematics been loitering about in the vestibule of the pedagogical edifice long enough now to be ready actually to enter it?

It is in the hope of contributing, though perhaps only a little, toward revealing the pedagogical possibilities that lie immediately before us, that the following recital of the details of a partially completed experiment is presented.

A few months ago a professional student, who was at the time a regular high-school teacher of mathematics, undertook the problem of systematically testing the *heuristic* as against the *expository* mode of teaching algebra to first-year high-school classes. His aim was to make the test as objective in character as possible. He chose for the experiment two of his first-year algebra classes that were, in his judgment, similarly circumstanced and nearly equal in ability. Confirmatory evidence of their equality of ability was furnished also by previous tests.

One class was taught by the heuristic plan as nearly as the teacher could administer it, and the other was taught entirely by the plan of direct exposition. To the first class the teacher told

nothing directly, but he questioned indirectly and worked out inductively what he wanted learned. He made it a point to have the mathematical truth come first over the lips of the learner. To the second class he told everything directly, illustrating copiously and with great care. Both at the beginning and at the close of the regular teaching period of fifty minutes he gave arithmetical drills or exercises, of exactly two minutes' duration, as a means of measuring the fatigue factor of the methods. The classes numbered twenty and twenty-three pupils respectively.

The beginning test of the first day was as follows: Each pupil began with 6 and wrote as many correct sums as he could, adding first 1, then 2, then 3, 4, 5, etc., during the two minutes. The final two-minute test the pupils began with 8, and added the natural numbers in succession as above. Records of "attempts" and "rights" were kept, that furnished the following class averages:

EXPERIMENT I

Method	Beginning	End
Expository	96.70 per cent	97.12 per cent
Heuristic	96.36 "	95.90 "

The numbers denote the class averages of the percentages of the "attempts" that were "rights."

The beginning test the second day consisted in writing, in exactly two minutes, as many as possible of the sums: $2+5$, $4+7$, $6+9$, $8+1$, $2+3$, $4+5$, etc., the first number being even, the second, odd; the next sum being the next even plus the next odd, etc. When 8 was reached the even series started over again with 2, and when 9 was reached the odd series started over with 1. The end test was the same in kind, but the pupils started with a different sum, as $4+3$, $6+5$, etc.

There were so few errors in these sums that only the *numbers of sums* set down ("attempts") were counted. When a pupil wrote down more sums in the end test than in the beginning, the number expressing the excess was given a positive sign and for deficiency of end results over beginning results a negative sign was written before the excess. These positive and negative

numbers were then added separately, and, finally the sums were subtracted, with the results:

EXPERIMENT II

Expository.....54− 5=49
Heuristic.....41−18=23

On the whole, then, while the class taught by the expository plan accumulated an excess sum of 49 in the end test, the class taught heuristically accumulated a total of only 23 in excess. Accordingly, both methods seemed stimulating, but the expository was the more so.

A third test was given a few weeks later in which for the fatigue test the beginning task was to start with 8, then add 6, then 7, then again 6, and again 7, and so on, for exactly two minutes. Only the sums were to be written down by pupils. In the end two-minute task the class was to begin with 6, then add 6, then 7, then again 6, and again 7, and so on as above. The class results were scored with the following outcome:

EXPERIMENT III

Expository.....49−29=20
Heuristic.....40−66=−26

From external evidence shown by past tests, the first experiment seemed to reveal less of the effect of practice than did either of the other two. Still the increase in the totals of the sums in subsequent experiments was not great enough to require allowance for it, since only differential effects were involved in the final results. The only "practice effects" shown were such as were fully accounted for by the consideration that pupils knew at once in the later experiments precisely what was expected of them.

The results of these three experiments show pretty clearly that the heuristic method is more fatiguing than the expository. In all three experiments the expository plan showed more work done in the end two-minute fatigue test than in the beginning test. Thus, the expository plan showed itself actually *stimulating* throughout. With the heuristic plan less work was accomplished by pupils in the final fatigue test than in the beginning in Experiments I and III. There were 26 fewer sums in the end period than in the beginning in Experiment III. In this same experiment the

expository put 20 more sums on record in the end than in the beginning test.

In Experiment II both methods put more sums on record in the end than in the beginning two-minute period (the expository 49 more, and the heuristic 23 more), and hence, while both methods appeared stimulating, the expository plan was the more so.

The results of the first experiment are however not sufficiently dependable. The unit of performance being very large the results are not sufficiently sensitive to reveal difference of performance clearly. Furthermore, the change of plan to the second experiment had not come sufficiently well under control to enable it to show the outcome accurately. It is believed, however, that the results of Experiment III are more than fairly satisfactory. The conclusion seems to be, so far as these meager data warrant a conclusion, that the heuristic method is more exhausting of the energy of pupils than the expository. So much so that it may even be concluded that the heuristic is positively exhausting, while the expository is positively stimulating. Under the heuristic plan the pupil does most of the work, and it naturally makes heavy drafts on his energy.

But is the gain worth the cost? Is the benefit to the pupil commensurate in some sense to the greater expenditure of energy?

The experimenter planned very careful tests covering the work gone over each fortnight while an experiment was in progress. The papers were graded with particular care according to his customary plan, and the class averages for the three experiments were as given here:

Method	1st Experiment	2d Experiment	3d Experiment
Expository	88.6 per cent	93.2 per cent	91.6 per cent
Heuristic	91.8 "	95.6 "	98.7 "

Clearly, then, the class taught by the heuristic method showed a stronger and a more enduring grasp of the work gone over. It held a 3 to 7 per cent higher average than the class taught by the expository plan throughout the fortnightly tests. If these results could be regarded as final, the case for the heuristic method would seem pretty strong. The results, however, need confirmation or

refutation, and any teacher in the actual progress of his work in first-year algebra would perform a service of real value by testing these conclusions more thoroughly. Evidently, the character of the class work has not been marred by these experiments, for the class averages show a general rise throughout the experiments.

This article is making no statement that the foregoing conclusions are final. It merely hopes to suggest an experiment that is capable of refinement into a practicable means of testing methods of teaching through their results. The case here looks decidedly strong for the heuristic plan for first-year algebra. The test has the merit of being highly objective. Only an accumulation of evidence such as the foregoing can however settle the question of general superiority of one of these plans over the other. Obviously, it is well worth while to ascertain from actual classroom results just what particular points of superiority the heuristic, or expository, or any other special method has for mathematical teaching, as well as to determine what sort of combination of special procedures will properly provide for training the young mathematician to work both with his fellows and independently. Perhaps teachers may yet devise a plan involving a limited amount of stimulating, considerable evaluating of the results of the stimuli, and a great deal of inspiringly independent thinking on the part of pupils. The foregoing results seem to enable us to glimpse some such ultimate possibility.

Is it too sanguine a hope that some high-school teacher who may read this paper may be prompted to try out thus objectively the particular relative merits of the foregoing and other much-talked-of methods of teaching the mathematical subjects? By the gradual accretion of pertinent experiences and experiments some degree of definiteness as to the characteristic values of special methods may be attained, and in the writer's opinion, even such definiteness is eminently desirable in high-school mathematics.

The testing plan itself, be it noted, has some merits that seem worthy of attention. Everyone knows that algebra pupils are in continual need of arithmetical reviews, and practice. The two-minute test periods furnish an opportunity systematically to supply the needed work without robbing the regular work of its scanty time allotment. Anything on which mechanical drill is felt to

be needed may well be selected for these two brief testing periods. Principles of algebra itself, that have been mastered and now need merely to be drilled upon, may very well be called into requisition here. The aim is to get material for these fatigue-testing periods that requires close concentration for a brief space of time. It is concentrated attention that fatigues, and it is fatigue that we wish to measure.

The fortnightly testing of the work gone over also furnishes the necessary testing of the degree of control of the subject-matter taught, without material disturbance of the regular classroom procedure. Possible fatigue and strength of grasp must both be measured before the worth of a method can be known, and the testing plan seems to comprise these two essential factors fairly well.

There is also the added benefit to the teaching that comes from planning definitely for a specified result, and then carrying the plan forward from day to day, and checking up regularly as the work advances. Better teaching will be done, more interest will be developed, greater satisfaction will be felt that something definite is being accomplished, and the measured results will give tone, zest, and guidance to the regular work as it progresses.

It would perhaps be too much to hope for a thousand, or even a hundred, school tests such as the one sketched above. Such a body of data would however furnish a means of putting the question of the relative merits of teaching methods in mathematics on a more rational and objective basis.

It does not seem too much, however, to request that teachers of high-school mathematics who are well enough equipped professionally to be able to vary their classroom routine, undertake to test the relative merits of two or three of their favorite methods, or to supplement the foregoing tests on the two methods here involved, and to make their results available to the profession through some of the standard school journals. If the scheme described above does not seem the best, it may at least serve to remind the reader of a better one and to stimulate him to apply his own scheme. In behalf of the much-needed service of making mathematical classroom practice more objective, and hence more truly professional, the busiest teacher can afford to contribute of his professional effort and skill.